	MIDTERM EXAM 2	
Name, Surname:	Department:	GRADE
Student No:	Course: Linear Algebra	
Signature:	Exam Date: 22/05/2019	

Choose 5 out of 6 problems. Each problem is worth equal points. Duration is 60 minutes.

1. Write YES or NO: Is the set of all vectors of the form $\begin{bmatrix} a \\ b \end{bmatrix}$, where $b = a^3$, a subspace of R^2? Show your work.

Solution: NO because $2\begin{bmatrix} 2 \\ 8 \end{bmatrix} = \begin{bmatrix} 4 \\ 16 \end{bmatrix}$ which is not in the form $b = a^3$.

2. If $\begin{bmatrix} 2 \\ 13 \\ 6 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$. Find c_1, c_2, c_3.

Solution: $\begin{bmatrix} 1 & 1 & 1 \\ 5 & 2 & 4 \\ -1 & 1 & 3 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 13 \\ 6 \end{bmatrix}$. $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 5 & 2 & 4 & 13 \\ -1 & 1 & 3 & 6 \end{array}\right]$. Bring into row echelon form to find that $\mathbf{b} = \mathbf{v}_1 - 2\mathbf{v}_2 + 3\mathbf{v}_3$.

3. Find a basis for the solution space of $A\mathbf{x} = \mathbf{0}$ where $A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 1 & 2 & 1 & 4 \\ 2 & 4 & 1 & 5 \end{pmatrix}$.

Solution: RREF of $A\mathbf{x} = \mathbf{0}$ is $\left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$. The solution satisfies $x_3 + 3x_4 = 0$ and $x_1 + 2x_2 + x_4 = 0$.
$x_2 = r$, $x_4 = s$.
$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2r - s \\ r \\ -3s \\ s \end{bmatrix} = r\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -1 \\ 0 \\ -3 \\ 1 \end{bmatrix}$. A basis is $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -3 \\ 1 \end{bmatrix}$.

4. Let $S = \{1 + t + t^3, t + t^2 - t^3, t - t^2 + t^3\}$? Write YES or NO: Are the vectors in S linearly independent in P_3? Show your work.

Solution: YES linearly independent. $c_1(1 + t + t^3) + c_2(t + t^2 - t^3) + c_3(t - t^2 + t^3) = 0$. This gives 4 equations: $c_1 = 0$, $c_2 + c_3 = 0$, $c_2 - c_3 = 0$, $-c_2 + c_3 = 0$. The only solution is $c_1 = c_2 = c_3 = 0$.

5. Find a basis for the subspace of R^4 which consists of the vectors of the form $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ with $a - 2c + d = 0$ and $b + 3c - 4d = 0$.

Solution: $\begin{bmatrix} 2c-d \\ -3c+4d \\ c \\ d \end{bmatrix} = c \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix} + d \begin{bmatrix} -1 \\ 4 \\ 0 \\ 1 \end{bmatrix}$. The basis vectors are $\begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 0 \\ 1 \end{bmatrix}$

6. Find all values of a for which the vectors $\begin{bmatrix} 1 \\ a \\ 0 \end{bmatrix}, \begin{bmatrix} a \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ DO NOT form a basis of R^3.

Solution: 3 vectors in R^3 form a basis if the determinant of the corresponding matrix is non-zero. $\begin{vmatrix} 1 & a & 0 \\ a & 0 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 1 - a^2$. Hence determinant is zero if $a = \pm 1$. The answer is $a = \pm 1$.